From the PHILOSOPHICAL MAGAZINE *for April* 1899.

On Opacity. *By* Professor OLIVER LODGE, *D.Sc., LL.D., F.R.S., President of the Physical Society**.

MY attention has recently been called to the subject of the transmission of electromagnetic waves by conducting dielectrics—in other words, to the opacity of imperfectly conducting material to light. The question arose when an attempt was being made to signal inductively through a stratum of earth or sea, how far the intervening layers of moderately conducting material were able to act as a screen; the question also arises in the transmission of Hertz waves through partial conductors, and again in the transparency of gold-leaf and other homogeneous substances to light.

The earliest treatment of such subjects is due of course to Clerk Maxwell thirty-four years ago, when, with unexampled genius, he laid down the fundamental laws for the propagation of electric waves in simple dielectrics, in crystalline media, and in conducting media. He also realised there was some strong analogy between the transmission of such waves through space and the transmission of pulses of current along a telegraph-wire. But naturally at that early date not every detail of the investigation was equally satisfactory and complete.

Since that time, and using Maxwell as a basis, several mathematicians have developed the theory further, and no one with more comprehensive thoroughness than Mr. Oliver Heaviside, who, as I have said before, has gone into these matters with extraordinarily clear and far vision. I may take the opportunity of calling or recalling to the notice of the Society, as well as of myself, some of the simpler developments of Mr. Heaviside's theory and manner of unifying phenomena and processes at first sight apparently different; but first I will deal with the better-known aspects of the subject.

Maxwell deals with the relation between conductivity and opacity in his Art. 798 and on practically to the end of that famous chapter xx. ('Electromagnetic Theory of Light'). He discriminates, though not very explicitly or obtrusively, between the two extreme cases, (1) when inductive capacity or electric inductivity is the dominant feature of the medium—when, for instance, it is a slightly conducting dielectric, and (2) the other extreme case, when conductivity is the predominant feature.

* Read before the Physical Society of London, being the Presidential Address for 1899.

The equation for the second case, that of predominant conductivity, is

$$\frac{d^2\mathrm{F}}{dx^2}=\frac{4\pi\mu}{\sigma}\frac{d\mathrm{F}}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

F being practically any vector representing the amplitude of the disturbance ; for since we need not trouble ourselves with geometrical considerations such as the oblique incidence of waves on a boundary &c., we are at liberty to write the ∇ merely as d/dx, taking the beam parallel and the incidence normal.

No examples are given by Maxwell of the solution of this equation, because it is obviously analogous to the ordinary heat diffusion fully treated by Fourier.

Suffice it for us to say that, taking F at the origin as represented by a simple harmonic disturbance $\mathrm{F}_0=e^{ipt}$, the solution of equation (1)

$$\frac{d^2\mathrm{F}}{dx^2}=\frac{4\pi\mu ip}{\sigma}\mathrm{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1')$$

is

$$\mathrm{F}=\mathrm{F}_0\, e^{-\mathrm{Q}x}=e^{-\mathrm{Q}x+ipt},$$

where

$$\mathrm{Q}=\sqrt{\left(\frac{4\pi\mu ip}{\sigma}\right)}=\sqrt{\frac{2\pi\mu p}{\sigma}}\,.\,(1+i)\,;$$

wherefore

$$\mathrm{F}=e^{-\left(\frac{2\pi\mu p}{\sigma}\right)^{\frac{1}{2}}x}\cos\left(pt-\left(\frac{2\pi\mu p}{\sigma}\right)^{\frac{1}{2}}x\right), \quad . \quad . \quad . \quad (2)$$

an equation which exhibits no true elastic wave propagation at a definite velocity, but a trailing and distorted progress, with every harmonic constituent going at a different pace, and dying out at a different rate ; in other words, the *diffusion* so well known in the case of the variable stage of heat-conduction through a slab.

In such conduction the gain of heat by any element whose heat capacity is $cp\,dx$ is proportional to the difference of the temperature gradient at its fore and aft surfaces, so that

$$cp\,dx\frac{d\theta}{dt}=d\,.\,k\frac{d\theta}{dx},$$

or, what is the same thing,

$$\frac{d^2\theta}{dx^2}=\frac{cp}{k}\frac{d\theta}{dt},$$

the same as the equation (1) above ; wherefore the constant cp/k, the *reciprocal* of the thermometric conductivity, takes the place of $4\pi\mu/\sigma$, that is, of electric conductivity; otherwise the heat solution is the same as (2). The 4π has come in from an unfortunate convention, but it is remarkable that the conductivity term is inverted. The reason of the inversion of this constant is that, whereas the *substance* conveys the heat waves, and by its conductivity aids their advance, the *æther* conveys the electric waves, and the substance only screens and opposes, reflects, or dissipates them.

This is the case applied to sea-water and low frequency by Mr. Whitehead in a paper which he gave to this Society in June 1897, being prompted thereto by the difficulty which Mr. Evershed and the Post Office had found in some trials of induction signalling at the Goodwin Sands between a coil round a ship at the surface and another coil submerged at a depth of 10 or 12 fathoms. It was suspected that the conductivity of the water mopped up a considerable proportion of the induced currents, and Mr. Whitehead's calculation tended, or was held to tend, to support that conclusion.

To the discussion Mr. Heaviside communicated what was apparently, as reported, a brief statement; but I learn that in reality it was a carefully written note of three pages, which recently he has been good enough to lend me a copy of. In that note he calls attention to a theory of the whole subject which in 1887 he had worked out and printed in his collected 'Electrical Papers,' but which has very likely been overlooked. It seems to me a pity that a note by Mr. Heaviside should have been so abridged in the reported discussion as to be practically useless; and I am permitted to quote it here as an appendix (p. 413).

Meanwhile, taking the diffusion case as applicable to sea-water with moderately low acoustic frequency, we see that the induction effect decreases geometrically with the thickness of the oceanic layer, and that the logarithmic decrement of the amplitude of the oscillation is $\sqrt{\left(\frac{2\pi\mu p}{\sigma}\right)}$, where σ is the specific resistance of sea-water and $p/2\pi$ is the frequency.

Mr. Evershed has measured σ and found it 2×10^{10} C.G.S., that is to say $2\times10^{10}\,\mu$ square centim. per second; so putting in this value and taking a frequency of 16 per second, the amplitude is reduced to $1/e$th of what its value would have been at the same distance in a perfect insulator, by a depth

$$\sqrt{\frac{\sigma}{2\pi\mu p}}=\sqrt{\left(\frac{2\times10^{10}\mu}{2\pi\mu\times2\pi\times16}\right)}=\sqrt{\frac{10^{10}}{320}}=\frac{10^{5}}{18}\text{ centim.}$$
$$=55\text{ metres.}$$

Four or five times this thickness of intervening sea would