HIGHER ALGEBRA

A SEQUEL TO

ELEMENTARY ALGEBRA FOR SCHOOLS

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PREFACE.

THE present work is intended as a sequel to our *Elementary Algebra for Schools*. The first few chapters are devoted to a fuller discussion of Ratio, Proportion, Variation, and the Progressions, which in the former work were treated in an elementary manner; and we have here introduced theorems and examples which are unsuitable for a first course of reading.

From this point the work covers ground for the most part new to the student, and enters upon subjects of special importance: these we have endeavoured to treat minutely and thoroughly, discussing both bookwork and examples with that fulness which we have always found necessary in our experience as teachers.

It has been our aim to discuss all the essential parts as completely as possible within the limits of a single volume, but in a few of the later chapters it has been impossible to find room for more than an introductory sketch; in all such cases our object has been to map out a suitable first course of reading, referring the student to special treatises for fuller information.

In the chapter on Permutations and Combinations we are much indebted to the Rev. W. A. Whitworth for permission to make use of some of the proofs given in his *Choice and Chance*. For many years we have used these proofs in our own teaching, and we are convinced that this

part of Algebra is made far more intelligible to the beginner by a system of common sense reasoning from first principles than by the proofs usually found in algebraical text-books.

The discussion of Convergency and Divergency of Series always presents great difficulty to the student on his first reading. The inherent difficulties of the subject are no doubt considerable, and these are increased by the place it has ordinarily occupied, and by the somewhat inadequate treatment it has hitherto received. Accordingly we have placed this section somewhat later than is usual; much thought has been bestowed on its general arrangement, and on the selection of suitable examples to illustrate the text; and we have endeavoured to make it more interesting and intelligible by previously introducing a short chapter on Limiting Values and Vanishing Fractions.

In the chapter on Summation of Series we have laid much stress on the "Method of Differences" and its wide and important applications. The basis of this method is a well-known formula in the Calculus of Finite Differences, which in the absence of a purely algebraical proof can hardly be considered admissible in a treatise on Algebra. The proof of the Finite Difference formula which we have given in Arts. 395, 396, we believe to be new and original, and the development of the Difference Method from this formula has enabled us to introduce many interesting types of series which have hitherto been relegated to a much later stage in the student's reading.

We have received able and material assistance in the chapter on Probability from the Rev. T. C. Simmons of Christ's College, Brecon, and our warmest thanks are due to him, both for his aid in criticising and improving the text, and for placing at our disposal several interesting and original problems.

It is hardly possible to read any modern treatise on Analytical Conics or Solid Geometry without some know-

ledge of Determinants and their applications. We have therefore given a brief elementary discussion of Determinants in Chapter XXXIII., in the hope that it may provide the student with a useful introductory course, and prepare him for a more complete study of the subject.

The last chapter contains all the most useful propositions in the Theory of Equations suitable for a first reading. The Theory of Equations follows so naturally on the study of Algebra that no apology is needed for here introducing propositions which usually find place in a separate treatise. In fact, a considerable part of Chapter XXXV. may be read with advantage at a much earlier stage, and may conveniently be studied before some of the harder sections of previous chapters.

It will be found that each chapter is as nearly as possible complete in itself, so that the order of their succession can be varied at the discretion of the teacher; but it is recommended that all sections marked with an asterisk should be reserved for a second reading.

In enumerating the sources from which we have derived assistance in the preparation of this work, there is one book to which it is difficult to say how far we are indebted. Todhunter's *Algebra for Schools and Colleges* has been the recognised English text-book for so long that it is hardly possible that any one writing a *text-book* on Algebra at the present day should not be largely influenced by it. At the same time, though for many years Todhunter's *Algebra* has been in constant use among our pupils, we have rarely adopted the order and arrangement there laid down; in many chapters we have found it expedient to make frequent use of alternative proofs; and we have always largely supplemented the text by manuscript notes. These notes, which now appear scattered throughout the present work, have been collected at different times during the last twenty

years, so that it is impossible to make definite acknowledgement in every case where assistance has been obtained from other writers. But speaking generally, our acknowledgements are chiefly due to the treatises of Schlömilch, Serret, and Laurent; and among English writers, besides Todhunter's *Algebra*, we have occasionally consulted the works of De Morgan, Colenso, Gross, and Chrystal.

To the Rev. J. Wolstenholme, D.Sc., Professor of Mathematics at the Royal Indian Engineering College, our thanks are due for his kindness in allowing us to select questions from his unique collection of problems; and the consequent gain to our later chapters we gratefully acknowledge.

It remains for us to express our thanks to our colleagues and friends who have so largely assisted us in reading and correcting the proof sheets; in particular we are indebted to the Rev. H. C. Watson of Clifton College for his kindness in revising the whole work, and for many valuable suggestions in every part of it.

May, 1887.

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PREFACE TO THE THIRD EDITION.

In this edition the text and examples are substantially the same as in previous editions, but a few articles have been recast, and all the examples have been verified again. We have also added a collection of three hundred Miscellaneous Examples which will be found useful for advanced students. These examples have been selected mainly but not exclusively from Scholarship or Senate House papers; much care has been taken to illustrate every part of the subject, and to fairly represent the principal University and Civil Service Examinations.

March, 1889.

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