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SOUTHERN FEDERAL UNIVERSITY

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**LECTURES
ON DIFFERENTIAL CALCULUS
OF FUNCTIONS OF ONE VARIABLE**

Textbook for students
of science and engineering

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The textbook contains lecture material for the first semester of the course on mathematical analysis and includes the following topics: the limit of a sequence, the limit of a function, continuous functions, differentiable functions (up to Taylor's formula, L'Hospital's rule, and the study of functions by differential calculus methods). A useful feature of the book is the possibility of studying the course material at the same time as viewing a set of 22 video lectures recorded by the author and available on youtube.com. Sections and subsections of the textbook are provided with information about the lecture number, the start time of the corresponding fragment and the duration of this fragment. In the electronic version of the textbook, this information is presented as hyperlinks, allowing reader to immediately view the required fragment of the lecture.

The textbook is intended for students specializing in science and engineering.

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