

P. G. Demidov Yaroslavl State University
International B. N. Delaunay Laboratory
Discrete and Computational Geometry

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on Discrete and Computational Geometry
July – August, 2012

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The International Delaunay Laboratory of Discrete and Computational Geometry introduces a new books series: the *Delaunay Library*. Its mission is the advancement of research and education in this field. Specifically, we propose to publish scientific monographs, collections of papers on current research, and textbooks whose primary topics are in the fields of discrete geometry, computational geometry, and computational topology. The Delaunay Laboratory is being funded by the Russian Government since 2011 and is lead by Herbert Edelsbrunner. It constitutes a concerted effort of leading international specialists to create a new world-class research and educational center in Discrete and Computational Geometry on the premises of the P. Demidov Yaroslavl State University. The introduction of the Delaunay Library is part of its educational and promotional activities. The Delaunay Laboratory entrusts the Editorial Board of the Delaunay Library – consisting of three Head Editors, *N. Dolbilen*, *H. Edelsbrunner*, and *A. Ivanov*, and nine Editors, *V. Buchstaber*, *V. Dolnikov*, *R. Karasev*, *V. Manturov*, *N. Moshchevitin*, *O. Musin*, *M. Nevskii*, *I. Sabitov*, and *M. Shtogrin* – with the development of the library. For a book to be included in this series, it must be approved by this board. Additional information on the series, including annotations of the books in preparation, can be found on the web page of the Laboratory (www.dcglab.uniyar.ac.ru).

Similar to the Laboratory, the series is named in honor of Boris Delaunay, a brilliant Russian geometer and one of the founders of modern discrete geometry. He was also a wonderful teacher and famous mountain climber. The Delaunay peak, more than four thousand meters high in the Altai mountains of Russia, is as well-known to alpinists as is the Delaunay triangulation to mathematicians. A stylized design of the peak created by Xixi Edelsbrunner can be seen on the front covers of the books in the series, and a hint of

the peak's contour is given by the letter Λ in the Laboratory logo designed by A. Akopyan and S. Sharov–Delaunay.

We open the series with the *First Yaroslavl Summer School on Discrete and Computational Geometry*, which contains lecture notes accompanying the short courses delivered in July and August of 2012. We hope this volume will be useful to all students and post graduates interested in the current state-of-the-art in this field. Currently, several additional volumes are in preparation. Among them is the *Delaunay Volume*, a collection of works by Boris Delaunay, selected and commented by his students N. Dolbilin and M. Shtogrin; the *Voronoi Volume*, a collection of papers by Georgii Voronoi, selected and commented by O. Musin; and a Russian translation of the textbook on *Computational Topology* by H. Edelsbrunner and J. L. Harer.

We are confident that the Delaunay Library becomes a valuable resource for getting acquainted with, studying, teaching, and doing research in modern discrete and computational geometry and related topics.

N. Dolbilin, H. Edelsbrunner, and A. Ivanov

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